SOLID STATE PHYSICS 1

- April 2008 -

Do not forget to write your full name and student number on each sheet.

Please use separate sheets for each of the four problems.

Planck's constant	$\hbar$	1.055×10^{-34} Js
Planck's constant	h	6.626×10^{-27} erg s
Boltzmann's constant	k_B	1.38×10^{-23} J K^{-1}
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
speed of light	c	3.0×10^{8} m s^{-1}
elementary charge	e	1.60×10^{-19} C
mass of the electron	m	9.11×10^{-31} kg
Bohr magneton	μ_B	9.27x10-24 J T^{-1}
Note also: 1 eV = 1.6019×10^{-12} erg		

I. The following questions should be answered very briefly (1-2 sentences) [2 points for each question]

a) In a metal, what is a plasmon? How can it be excited?

b) Why do nanostructures have different properties from bulk crystals?

c) What is meant by the cohesive energy of a solid?

d) What is thermopower ? What can we learn from it regarding the charge carriers in a semiconductor?

e) Define the magnetic susceptibility per unit volume in CGS or SI. What are substances with negative susceptibility called?

f) What is the necessary condition for a structural phase transition to occur in a crystal?

g) What determines the electrical resistivity of Cu at room temperature and at 4 K?

h) What happens when a type 1 superconductor is immerged into a homogeneous magnetic field ($H<H_c$; $T<T_c$)? How are H_c and T_c related?

i) What special property must a crystal have to be called ferroelectric?

j) Apply Hund's rule to find the angular momentum quantum number, the spin quantum number and the total angular + spin momentum quantum number for Pr^{3+} in the configuration $4f^2 5s^2 p^6$. Give also the spectroscopic notation of the ground state.

II. (a) Name the three most important probes used in diffraction experiments on crystals. What is the one essential condition they must all satisfy? [2 points]

(b) Calcium crystallizes in a face-centred cubic unit cell with a=0.556 nm. How many atoms does the unit cell contain? How many nearest neighbours does each atom have? [1 point]

(c) Calculate the distance between the 111 planes in a crystal of Ca. Repeat the calculation for the 222 planes. Which planes are closer? [2 points]

(d) Derive Bragg's law for the diffraction of radiation by a three dimensional crystal. [2 points]

(e) At what Bragg angle for diffraction from the 111 planes occur if one uses filtered Cu K_α radiation? The wavelength of the radiation is λ=0.15418 nm. [1 points]

(f) Repeat the calculation for the second order reflection of the 111 planes and for the first order reflection of the 222 planes. What conclusion can be made? [2 points]

III. (a) Explain what is meant by a phonon, and how they are useful for considering the dynamics and thermal properties of crystals. (No formulae, just description) [2 points]

(b) Discuss the interactions that are possible between phonons, and what causes them. [2 points]

(c) The coefficient of thermal conductivity of an insulator is described by the equation

$$\kappa = \frac{1}{3}\lambda v_s c,$$

where λ is the mean free path of the phonon, v_s is the velocity of sound, and c is the specific heat capacity of the material per unit volume.

The heat capacity of an insulator at low temperatures is given by

$$C = \frac{12\pi^4}{5} N k_B \left(\frac{T}{\theta_D}\right)^3,$$

where N is the number of atoms in the sample, k_B is Boltzmann's constant, T is the temperature, and θ_D is the Debye temperature. At high temperatures, C tends towards $3Nk_B$.

The conventional unit cell of diamond contains eight carbon atoms and has a lattice parameter of 0.35567 nm. The Debye temperature of diamond is 2230 K. At 3.75 K, the coefficient of thermal conductivity of diamond is 10 $Wm^{-1}K^{-1}$, and the velocity of sound is 9200 ms^{-1}.

i. What is the mean free path of the phonons in the material at this temperature? [2 points]

ii. As the temperature is raised, how would you expect the mean free path of phonons to change? [2 points]

iii. How would you expect the thermal conductivity to change as the temperature is raised? [2 points]

IV. The mean energy of an electron in a metal according to the free electron model is

$\bar{E} = (3/5)\, E_F$

where the Fermi energy is given by $E_F = \frac{\hbar^2 (3\pi^2 n)^{2/3}}{2m}$ and n and m are the density and the mass of free electrons respectively.

(a) Using the result dE = - p dV, where E is the total energy of electrons in volume V, derive an expression for the pressure p of the free electron gas. [2 points]

(b) Hence show, using the appropriate thermodynamic definition, that the chemical potential (the Gibbs free energy per electron) at T=0 is equal to E_F. [2 points]

(c) Calculate the chemical potential for the free electrons in pure silver, assuming one valence electron is made available per primitive cell, the volume of which is 1.7×10^{-29} m^3. [1 point]

(d) By adding divalent magnesium as an impurity as a concentration of one atom per million atoms of silver, by how much is the Fermi energy altered? [2 points]

(e) Silicon is an intrinsic tetravalent semiconductor with a band gap of about 1.1 eV. Discuss and explain the change in chemical potential of the electrons brought about by adding pentavalent or tetravalent impurities at a concentration of one atom per million atoms of silicon. Contrast this behaviour with the case of the silver-magnesium alloy just considered. [3 points]